❖ *Quantifiers and Variables: Translation and Semantics* ❖

5.4. Quantifiers and Variables

Quantifiers such as "all" and "some" are the last new bit of logical form. But as we'll see here and in the sections to follow, quantified sentences prove to be the most complex examples of logical form so far. So we begin by sharpening our understanding of them through simple English examples of what quantifiers are – and what they're not.

1. What Quantifiers Are Not: Names Revisited. Having mastered translation of proper names into the formal language, it is tempting to sweep other phrases under the umbrella of proper names, translating them the same way. The following two sentences, for instance, seem nicely parallel.[1]

(1) *Methods of Logic* is an interesting book.
(2) Something is an interesting book.

With "something" acting as the grammatical subject of (2), just as the proper name "*Methods of Logic*" does in (1), it could seem sensible to translate "something" by a name letter.

But the parallel breaks down on further analysis.

(1) *Methods of Logic* is an interesting book.
(3) *Methods of Logic* is interesting, and *Methods of Logic* is a book.

(2) Something is an interesting book.
(4) Something is interesting, and something is a book.

With proper names we made a practice of translating 'stacked up' predicates – like "is an interesting book" – as a conjunction in disguise. That treats (1) and (3) as equivalent in (logical) meaning. And that looks like the right result: (1) and (3) mean the same thing; and whenever one of these sentences is true, the other is as well.

[1] Adapting an example from (Quine 1982: 138).

But (2) and (4) aren't true in exactly the same situations.

(2) Something is an interesting book.
(4) Something is interesting, and something is a book.

In a universe containing just two objects – an interesting professor and a boring book – (4) would be true, but (2) would be false.

If that point seems too subtle, the next two sentences draw the same distinction more boldly.

(5) Something is a non-feline feline.
(6) Something is non-feline, and something is feline.

What (6) reports certainly seems possible – indeed, the actual world is a situation where something is non-feline (say, the book *Methods of Logic*) and something is feline (say, Neko). But what (5) reports seems impossible. Sentence (5) looks like a flat-out contradiction, false in every possible situation, including the actual one. So the actual world shows that (5) and (6) aren't true in all the same situations; and (predictably) the two sentences don't seem to mean the same thing.

By comparison: Sentences (7) and (8), using the proper name "Rex," do seem logically equivalent. (They both look like contradictions.)

(7) Rex is a non-feline feline.
(8) Rex is non-feline, and Rex is feline.

This illustrates that a quantifier such as "something," even when appearing as the subject of an English sentence, does **not** behave logically like a proper name. That's why we resist translating English quantifiers into name letters.

(We could make the same case in terms of truth and validity: given the obvious importance of truth and falsehood to validity, and the clear differences in truth and falsehood between Sentences (5) and (6), our formal language had better have a way of showing the difference between such sentences – when a sentence is talking about the same object(s) throughout, like (5), and when not, like (6). But as long as we translate quantifiers as names, that difference will be papered over – as examples such as (7) and (8) make clear. So again: we resist translating quantifiers as names.)

2. Quantifiers and Variables. Further examples steer us toward a proper translation of sentences with quantifiers. Suppose, first, that a materialist philosopher of mind makes the following **universal** claim.

(9) **Everything in the universe is a physical object**.

The predicate "is a physical object" is translated by a predicate letter.

G: ____ is a physical object

We still lack a way of talking formally about *every* object in the universe. But a somewhat wordy rephrasing of Sentence (9) offers a clue.

(10) **For every object in the universe, the following holds true of it: *it is a physical object*.**

Put this way, quantified sentence (9) has two components: (i) the mini-sentence "it is a physical object" on the right, and (ii) a quantifier phrase applying that mini-sentence to every object in the universe.

Now the mini-sentence "it is a physical object" resembles familiar English sentences such as "Plato is a physical object" which pose no obstacle to translation. And the word "it" does act in many respects like the proper name "Plato": both words can fill the blank in a predicate, yielding a sentence of English; and both serve to refer to some object. So here again we may be tempted to translate the term like a proper name.

But there's an important difference here as well. The name "Plato" is a **proper** name precisely because it always points to the same object. By contrast, the word "it" refers to different things at different times. So I can point to a chair and say "It is made of wood," then point to a bronze sphere and say "It is not made of wood". In so doing I have uttered no contradiction – since "it" in the two sentences pointed at different things. By contrast: if I say "Plato is from Pennsylvania" and "Plato is not from Pennsylvania" I have contradicted myself – since a proper name like "Plato" refers consistently to the same object.

Because "it" can vary in what it points at, the word only refers successfully with outside help – a pointing finger, or a context where one point-worthy

object is especially prominent. By varying the context or target of the finger, we vary the reference of "it". By contrast, the proper name "Plato" refers to Plato even when he's well out of eyeshot.

For that reason an "it" sentence of English without such outside help **fails to make a complete claim**. If you find a scrap of paper in a field with the sentence "It is from Pennsylvania" written on it, you don't know what claim is being made – not even if you know the geographical origins of every object in the world. But a scrap of paper reading "Plato is from Pennsylvania" on its own expresses a complete sentence, capable of truth or falsehood. (In fact it's false.) An "it" sentence isn't the sort of complete-claim-maker we find in a sentence using a proper name.

To highlight these differences, we resist translating "it" (and related terms like "this" and "that") as name letters. Instead we add new pointing symbols to the formal language: lower-case letters "a" through "z". These are the **variables**.

> **Variables**: lower-case letters **a** through **z**

In terms of sentence construction, variables show up in the same locations as proper names: after a predicate letter. So using the previous translation table we translate the ambiguous English mini-sentence "it is a material object" as follows.

> **G**: ____ is a physical object
>
> **It is a physical object**.
>
> **Gx**

This takes our universal sentence part-way into formal translation.

> **For every object in the universe, the following holds true of it:**
> ***it is a physical object***
>
> **For every object, *x*, the following holds of *x*: G*x***

We then need formal symbols for universal terms like "every" and "all". We will express universal terms formally by the **universal quantifier symbol** "∀". (When reading this symbol aloud, it's pronounced "universal".)

Combining a universal quantifier symbol with a variable yields a universal quantifier phrase – or "**universal quantifier**" for short – which means: "for every object, x, the following holds of x".

∀x

Now we can translate the entire English sentence.

G: ____ is a physical object.

(10) **Everything in the universe is a physical object**.

(For every object, *x*, the following holds of *x*: *x* is a physical object)

(11) **∀x Gx**

Next, suppose a dualist philosopher of mind disputes this materialist claim, holding instead that while some things in the universe are physical objects (tables and chairs), others (particularly minds or souls) are not.

(12) **Some things in the universe are physical objects,**
but some things in the universe are not physical objects.

That dualist claim is a *conjunction*. The left half is

(12a) Some things in the universe are physical objects

while the right half is

(12b) Some things in the universe are not physical objects.

Following a tradition reaching back to Aristotle, we interpret "**some**" to mean: **at least one**.

Rephrasing each of these sentences in the same wordy fashion as before, the left sentence reads as follows.

(12a) Some things in the universe are physical objects

For some (at least one) object, *x*, the following holds of *x*:
***x* is a physical object**

Employing the same translation table, the translation begins like this.

G: ____ is a physical object.

For some (at least one) object, *x*, the following holds of *x*: G*x*

As a formal means of expressing "some," we introduce the **existential quantifier symbol** "∃" (pronounced "existential"). Like the universal, the existential quantifier symbol combines with a variable to make an existential quantifier phrase, or "**existential quantifier**" for short.

We then translate the left sentence in the dualist conjunction like so.

(12a) Some things in the universe are physical objects

(13a) **∃x Gx**

And the dualist's claim in its entirety translates as follows.[2]

(12) Some things in the universe are physical objects,
but some things in the universe are not physical objects.

(13) **(∃x Gx ∧ ∃x ~Gx)**

Finally, suppose an idealist philosopher makes the following claim.

(14) **Nothing in the universe is a physical object**.

[2] This sentence illustrates again the difference stressed earlier: that the object(s) said to be physical and those claimed to be non-physical are not here said to be the *same* object(s). One might suppose different variables are called for – say, "x" and "y" – to express that difference. But as we'll see later ("5.6. Construction Revisited: Sentences, Formulas, and Binding"), different variables aren't in fact needed here to express that different objects are intended.

Do we need a third quantifier symbol to express "nothing"? While we could introduce a third quantifier symbol for this purpose, it turns out we can express "nothing" in our formal language using just symbols we have already.

Intuitively, the sentence "Nothing in the universe is a physical object" means the same as "Not (even) one thing in the universe is a physical object". So a "nothing" sentence can be read as the **denial of an existential sentence**.

G: ____ is a physical object.

Nothing in the universe is a physical object.

(15) **~∃x Gx**

As we will explore later, there are different, but semantically equivalent, ways of expressing all of these quantified sentences in the formal language.